Using tiling theory to generate and classify weaving patterns with beads

Gwen Fisher, beAd Infinitum, Sunnyvale, CA
gwen@beadinfinitum.com

Blake Mellor, Loyola Marymount University, Los Angeles, CA
blake.mellor@lmu.edu

Presented at the Joint Mathematics Meetings, Boston, MA, Jan 5, 2012

Abstract
Tilings of the plane, especially periodic tilings, can be used as the basis for flat bead weaving patterns called angle weaves. We describe specific ways to create infinitely many intricate and beautiful angle weaves from periodic tilings, by placing beads on or near the vertices or edges of a tiling and weaving them together with thread. We introduce the notion of star tilings and their associated angle weaves. We explain how many angle weaves can be generated in different ways, and we use these results to design graphic illustrations of many layered patterns. Lastly, we prove that there are infinitely many angle weaves, and we describe necessary and sufficient conditions for when a particular tiling of the plane will induce an angle weave.

Bead weavers create a wide variety of designs by joining beads with needle and thread, including flat weaves that resemble woven fabric as in Figure 1. Each bead in the weave is held in place by thread passing through its hole and the holes of neighboring beads at both ends. Loops of beads connect together to make fabric that can be flat or curved. Patches of beaded fabric are commonly used to make jewelry, especially bracelets and necklaces.

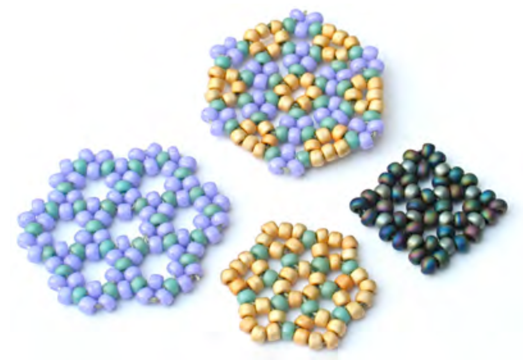

Figure 1: Flat angle weaves based on repeating tilings of the plane (clockwise from top): Snow Star with only vertex beads, super RAW, hexagon angle weave, Archimedes' Star with only vertex beads (See also across-edge weaves)

Flat angle weaves are arrangements of loops of beads that form flat patches that can be repeated to cover arbitrarily large regions. We describe several classes of angle weaves (see Figures 2, 3 and 4).

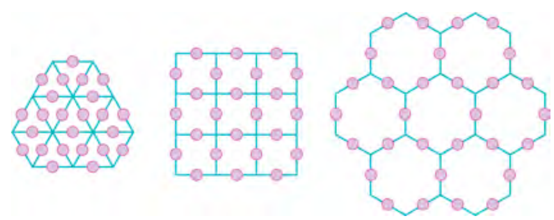

Figure 2: The regular tilings with one bead on each edge: triangle weave (3^6), right angle weave (4^4), and hexagon angle weave (6^3)

Figure 3: Examples of edge-only angle weaves: triangle weave (3^6), right angle weave (4^4), and hexagon angle weave (6^3) with fire-polished 4mm beads

Figure 4: Edge-and-cover angle weaves for (3^6), (4^4), (6^3)

To create a *star tiling*, from an initial polygonal tiling, we place stars at each vertex of the tiling. Examples of this transformation on the three regular tilings are shown in Figure 5.

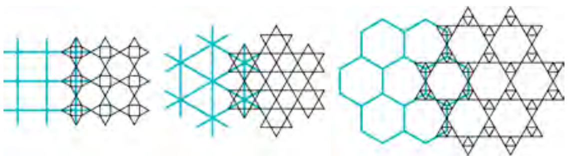

Figure 5: Star tilings of (4^4), (3^6), and (6^3): Kepler's Star, David's Star, and Archimedes' Star

A *star weave* is generated by first transforming a tiling into a star tiling, and then transforming the star tiling into an angle weave by placing one or more beads on each edge and/or each vertex of the star tiling. Figure 6 shows the Kepler's Star tiling with beads on every edge (left), with beads on every vertex and edge (center), and beads on every vertex (right).

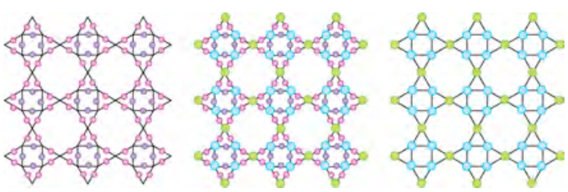

Figure 6: Kepler's Star with beads on edges only, vertices and edges, and vertices only (super RAW)

Figure 7: Kepler's Star Bracelet (vertices and edges) with size 11° and 15° seed beads

We prove several equivalences among different angle weaves generated from a tiling; one of these is illustrated in Figure 8.

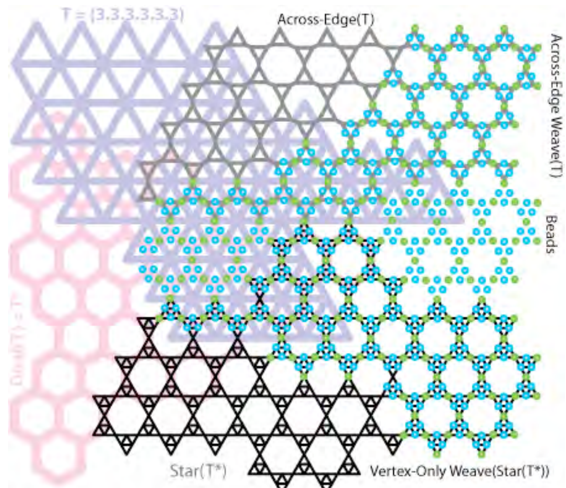

Figure 8: Theorem 2 with $T = (3^6)$

Finally, we prove that most tilings (specifically, all *normal* tilings, as defined by Grünbaum and Shephard) can be used to create angle weaves.

References

* Fisher, G., *Hexagon angle weave beads* (video), www.youtube.com/watch?v=6e9eV1c82VY, Accessed November 30, 2011
* Fisher, G., "Kepler's Star: A quick and easy flat weave" beAd Infinitum, Long Beach, CA, 2008
* Fisher, G., *Animated How to Weave Super Right Angle Weave with Beads* (video) www.youtube.com/watch?v=t5ENWHfF8JU, Accessed November 30, 2011
* Fisher, G., "Archimedes Star: A flat weave for bracelets and pendants" Beadwork Magazine, August/September 2009, Volume 12, Number 5
* Grünbaum, B. and Shephard, G.C., *Tilings and Patterns*, W.H. Freeman and Co., New York, 1987